# A residual stress analysis program using a Matlab GUI on an autofrettaged compound cylinder [†]

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**Abstract**

A program for the residual stress analysis of an autofrettaged compound cylinder is designed using a Matlab graphical user interface (GUI) and program design technique. The high-pressure vessels are autofrettaged in order to increase their operating pressure and fatigue life. An autofrettage process causes plastic expansion of the inner section of the cylinder, adding residual compressive stress to the bore after relaxation. Such a compound cylinder is produced via a shrink-fit procedure that incorporates a monobloc tube that has previously undergone autofrettage. This paper presents a simple and visual tool to calculate the residual stress and describe the distribution of residual stress for both the elastic-perfectly plastic model and the strain-hardening model.

*Keywords*: Matlab; GUI; Compound cylinder; von-Mises yield criterion; Strain-hardening; Autofrettage model

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## 1. Introduction

Matlab is a high performance, interactive software package used for scientific and engineering computations. It is popular in the control community and offers excellent performance qualities for designing regulation algorithms. This makes it the best candidate for accomplishing the objective we have fostered. GUIDE is a Matlab graphics user interface (GUI) design environment. It stores GUIs in two files, both of which are generated the first time GUI is saved or run. One file is a .fig file that contains a complete description of the GUI Fig. layout and the components of the GUI. The other file is an .m file that contains the code, which controls the GUI. Calculations and graphical presentations by means of these tools

can be displayed in a convenient manner.

The autofrettage process is a practical method for increasing the elastic-carrying capacity and fatigue life of a thick-walled cylinder, such as a cannon or a high-pressure tubular reactor [1]. There have been several investigative papers on the thick-wall cylinder and residual stress analysis. In 1977, Kim and Lee [2] investigated an autofrettage elastic-plastic analysis. In 2007, Lee and Park [3-5] studied and analyzed the Bauschinger effect about a compound cylinder. Swardt and Andrews [6] studied the stress analysis of autofrettaged midwall-cooled compound gun tubes in 2006.

The main goal of this paper is to present a Matlab GUI program for an autofrettaged compound cylinder. This program can provide a rapid tool to process data so that stress calculations can be done easily and stress distributions can be displayed intuitively. The stress analysis is performed on an autofrettaged compound cylinder with an elastic-perfectly plastic model

and strain-hardening model.

## 2. GUI development and program overview

The objective of this project is to design, build, and implement a residual stress-calculating GUI that is based on the study of Lee and Park [3-5]. Normally, a GUI has two functions: (1) A basic function and (2) an additional function. The basic function is a function of the Matlab program itself, allowing users to select which functions to create by choosing the components. In this study, the developed GUI program must be able to perform the following basic functions:

1. Display basic information, such as geometry shape and material property
2. Input the unknown parameters that are mentioned in former equations
3. Obtain visual results and graphs and display them in the appointed regions.

Additional functions are made by the Matlab GUI user. These are functions that are created to deepen the understanding of the basic functions and reduce the error input. The GUI program in this study should also be able to perform the following additional functions:

1. Show what each of the parameters stands for in the parameters menu
2. Pop error message windows automatically when the wrong parameter value is inputted
3. Store the calculation results
4. Exit the program on its interface.

The flowchart for a residual stress analysis program is presented as Fig. 1. This flowchart shows the processes in a more detailed diagram. This program includes three important calculations: two typical autofrettage models and a shrink fit analysis of the compound cylinder. Each of these parts will be explained shortly.

To start programming the GUI, the programmer must have a definite structure of the GUI in mind and know what the GUI needs to accomplish. The figures below will explain these. Fig. 2 represents the main structure of an interface menu, specifically, the .fig file that is controlled by an .m file. It includes four menus: (1) material menu, (2) parameters menu, (3) results menu, and (4) graphics window.

### 2.1 Material menu

The Material menu is divided into three parts: (1) geometry shape, (2) material property, and (3) stress
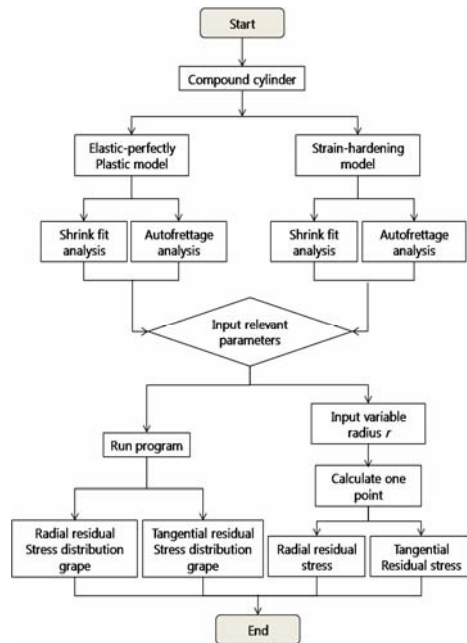


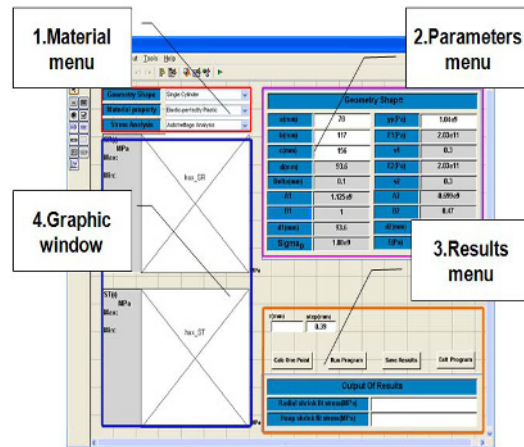Fig. 1. Flowchart for a residual stress analysis program.



Fig. 2. Main structure of an interface menu.

analysis parts. They are prepared using three drop-down menus in the GUI Fig. layout. Geometry shape is a compound cylinder. The material property includes two kinds of model: the elastic-perfectly plastic model and the strain-hardening model. The stress analysis consists of a with shrink fit analysis and autofrettage analysis. This program mainly performs research on the residual stress of the autofrettage compound cylinder in two typical autofrettage models: the elastic-perfectly plastic model and the strain hardening model. A theoretical analysis was con-

Fig. 3. The parameters menu.



(a) The parameter message

ducted using stress equations proposed by Park et al. [3-5].

### 2.2 Parameters menu

The parameters menu is divided into 18 input parameter fields that contain editable text in the GUI Fig. layout. All of the parameters are defined in the .m file. The parameters of the interface are edited according to the equations found in Lee and Park's (3-6) paper. The values can be inputted via the keyboard. In these fields, if the parameter edit background color is white, that parameter is used and a corresponding value can be inputted. When the parameter edit background color is gray, that parameter is not used.

Two of the additional functions have been implemented as shown in Fig. 4. The interface has many parameters and does not have enough space with which to show the definitions of each parameter. The air bubble presentation function of the Matlab software explains what each of the parameters signifies. When a mouse is used to click the parameter with properties that require further clarification, the parameter displays, as shown in Fig. 4(a), along with other descriptions. All of the parameters are defined in the program. Once any of the input is negative, an error window will display, as shown in Fig. 4(b).

### 2.3 The results menu and the graphics window

If we input the right values in the parameters menu, we can obtain the numerical value of the resulting radial residual stress and the tangential residual stress in the results menu. Fig. 5 shows the results menu, and includes the variable radius input window, step input window, some push buttons, and the results output window. The variable radius value must be between the values for the inner outer radii. The step



(b) An error message

Fig. 4. Additional functions of the program in the parameters menu. (a) The parameter message (b) An error message.



Fig. 5. The interface of the results menu displayed by Matlab.

denotes the distance between two points on the radius. The smaller the step input is, the more accurate the analysis results are. Two other additional functions are shown in Fig. 6. All of the parameters can be saved in two formats: (1) as a MAT file, or (2) as a .txt file. By pressing the "Exit Program" button, the program closes.

Stress distribution pictures can also be shown in the graphics window, as shown in Fig. 7. This paper used a Matlab p-color function to draw the pictures and used different colors to describe the stress distribution. The program also adopted the 1/4 model to show the stress distribution because the cylinder was symmet-
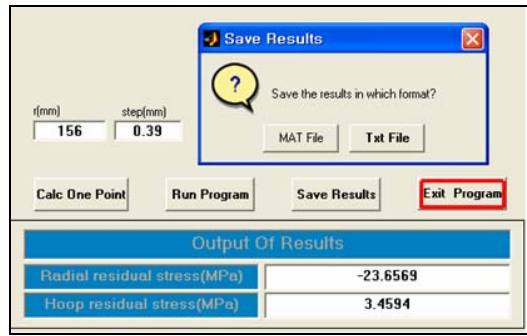
Fig. 6. Additional functions of the program in the results menu.
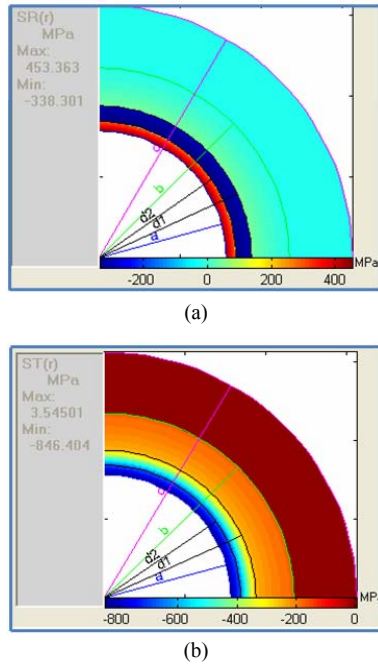


(a)



(b)

Fig. 7. The interface of the graphs menu displayed by Matlab (a) Radial residual stress distribution plot (b) Tangential residual stress distribution plot.

rical. The color changes under different parameters conditions. For example, Fig. 7(a) expresses the radial residual stress distribution plot while Fig. 7(b) expresses the tangential residual stress distribution plot. In other words, the values expressed by each color will change.

## 3. Analysis procedures [3-5]

The analytical formulation on the autofrettaged compound cylinder was adopted from the authors' results from previous studies [3-5].
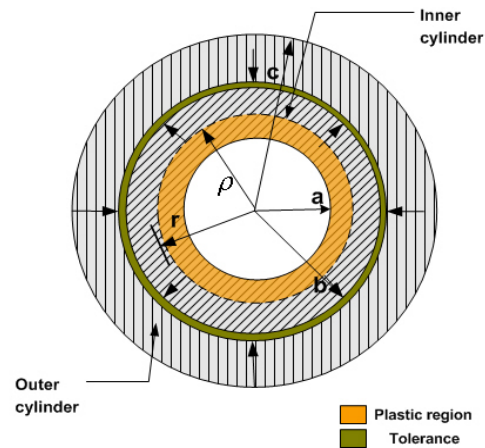


Fig. 8. A partially autofrettaged compound cylinder.

### 3.1 Analysis procedures on the elastic-perfectly plastic model

The radial residual stress and tangential residual stress are in the plastic region (a≤r≤ρ).

$\sigma_r^{p1}$　Eq. (A1) and　$\sigma_\theta^{p1}$　Eq. (A2)

Subscripts 'r' and 'θ' represent the radial direction and tangential direction, respectively.

The radial residual stress and tangential residual stress are in the elastic region (ρ≤r≤c).

$\sigma_r^{p2}$　Eq. (A3) and　$\sigma_\theta^{p2}$　Eq. (A4)

The elastic recovery stresses are given using the Lame equation:

$\sigma_r^e$　Eq. (A5) and　$\sigma_\theta^e$　Eq. (A6)

In this case, the compound cylinder was made via a shrink fit process. The inner radius of the outer cylinder may be made smaller than the outer radius of the inner cylinder. The cylinders were assembled after the outer cylinder was heated and inner cylinder was cooled. The stress distribution of cylinders as a result of a shrink fit can be calculated through equations (A7)-(A10).

The residual stress can be expressed through equations (A11)-(A12).

### 3.2 Analysis procedures on the strain-hardening model

Loading stress analysis in the plastic zone (a≤r≤ρ_L).
$\sigma_r^{L1}$　Eq. (A13) and　$\sigma_\theta^{L1}$　Eq. (A14)

Loading stress analysis in the elastic zone (ρ_L≤r≤ c).
$\sigma_r^{L2}$　Eq. (A15) and　$\sigma_\theta^{L2}$　Eq. (A16)

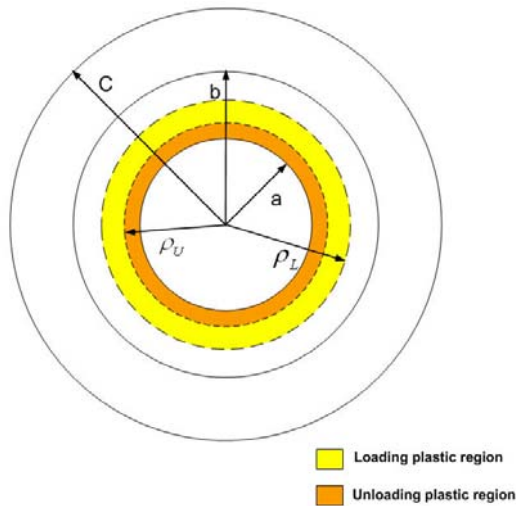Unloading stress analysis in the plastic zone (a≤r≤

Fig. 9. The loading and unloading plastic regions of the auto-frettaged compound cylinder.

$\rho_U$).

$\sigma_r^{U1}$  Eq. (A17) and  $\sigma_\theta^{U1}$  Eq. (A18)

Unloading stress analysis in the elastic zone ($\rho_U \leqslant r \leqslant c$).

$\sigma_r^{U2}$  Eq. (A19) and  $\sigma_\theta^{U2}$  Eq. (A20)

Shrink fit analysis is referred to in equations (A7)-(A10).

The residual stress can be expressed through equations (A21)-(A22).

## 4. The operation of the GUI

Once a user launches the stressTool program by Matlab, it will display as shown in Fig. 10. The operating procedures initially display. A user must first select the calculation condition from the material menu, as shown in Fig. 11. The user must then input the correct calculation parameter values into the parameters menu, as shown in Fig. 12. The user can run the program by clicking the "Run program" button and obtain the stress distribution graphs, as shown in Fig. 13. Finally, the user can input values of r at an arbitrary location by clicking the "Calculate one
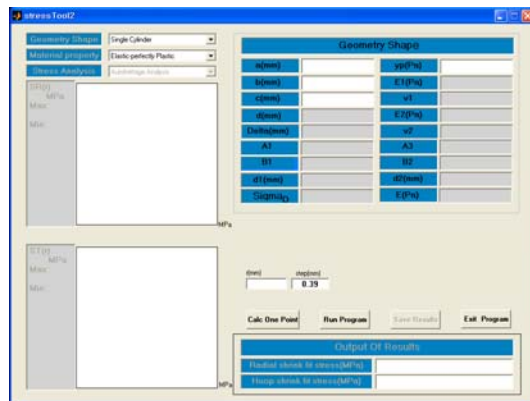


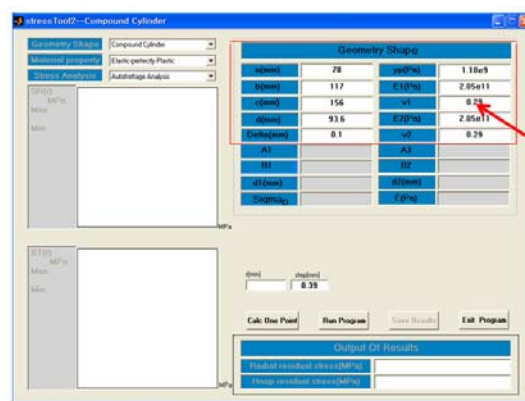Fig. 10. The initial GUI when launched.



Fig. 12. Inputting parameters into the available windows.
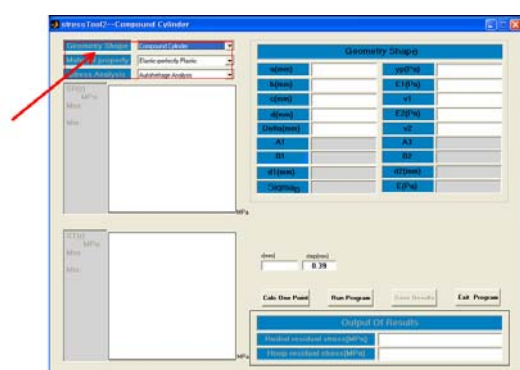


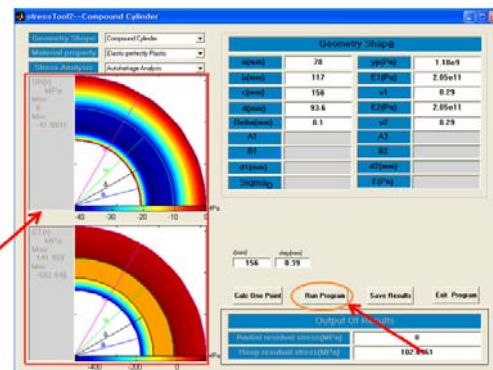Fig. 11. The user selects a calculation condition.



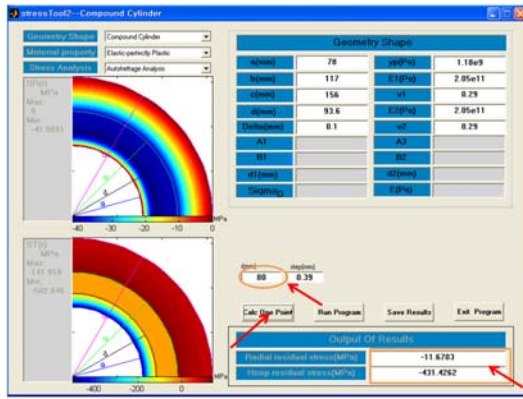Fig. 13. Running the program and plotting the graphs.
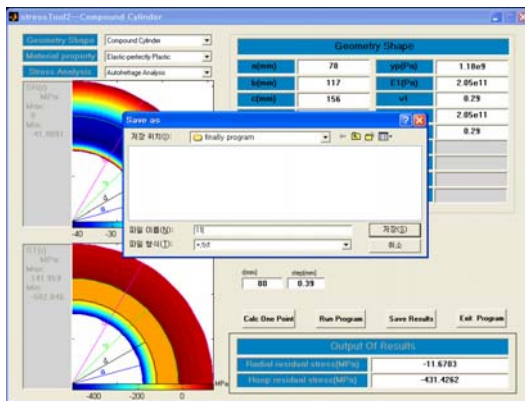
Fig. 14. Calculating the results for one point.



Fig. 15. Saving the file before exiting the program.

point" button. The results will be calculated and shown in the results menu. This is shown in Fig. 14. Fig. 15 shows the save file window and Fig. 16 shows the text file window.

## 5 Conclusions

In this paper, a Matlab-based GUI for an autofrettaged compound cylinder was developed. The basic and additional functions are the main functions of the GUI. The former can be used to create the main body frame for expedient processing. Its major functions include displaying basic information, enabling input parameters, and obtaining results. The latter plays a supplementary role to the basic functions and is used to show the descriptions of parameters and error messages, and store results.

The GUI of the Matlab program for an autofrettaged compound cylinder was developed and implemented successfully. This program provides a rapid method for processing data. In addition, stress
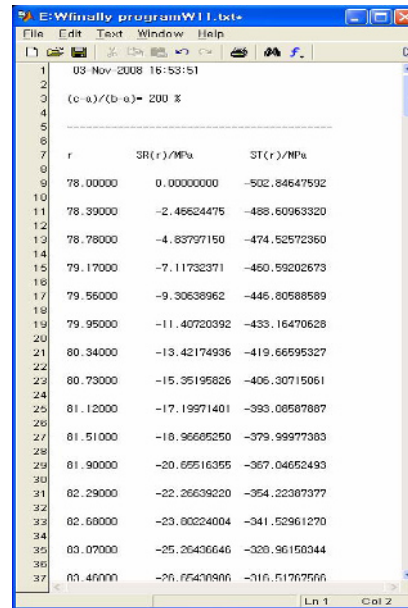


Fig. 16. The text file used to save all of the results.

calculations can be done easily and stress distributions are displayed intuitively.

## Acknowledgments

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## Appendix

$$\sigma_r^{p1} = \sigma_Y \left[ \ln \frac{r}{\rho} - \frac{1}{2}\left(1 - \frac{\rho^2}{c^2}\right) \right] \tag{A1}$$

$$\sigma_\theta^{p1} = \sigma_Y \left[ \ln \frac{r}{\rho} + \frac{1}{2}\left(1 + \frac{\rho^2}{c^2}\right) \right] \tag{A2}$$

$$\sigma_r^{p2} = \sigma_Y \frac{\rho^2}{2c^2}\left(1 - \frac{c^2}{r^2}\right) \tag{A3}$$

$$\sigma_\theta^{p2} = \sigma_Y \frac{\rho^2}{2c^2}\left(1 + \frac{c^2}{r^2}\right) \tag{A4}$$

$$\sigma_r^{e} = -\sigma_Y \left[ \ln \frac{\rho}{a} + \frac{1}{2}\left(1 - \frac{\rho^2}{c^2}\right) \right] \frac{a^2}{c^2 - a^2}\left(1 - \frac{c^2}{r^2}\right) \tag{A5}$$

$$\sigma_\theta^{e} = -\sigma_Y \left[ \ln \frac{\rho}{a} + \frac{1}{2}\left(1 - \frac{\rho^2}{c^2}\right) \right] \frac{a^2}{c^2 - a^2}\left(1 + \frac{c^2}{r^2}\right) \tag{A6}$$

$$\sigma_r^{s1} = -\frac{E\delta}{2b} \frac{c^2 - b^2}{c^2 - a^2}\left(1 - \frac{a^2}{r^2}\right) \tag{A7}$$

$$\sigma_\theta^{s1} = -\frac{E\delta}{2b} \frac{c^2 - b^2}{c^2 - a^2}\left(1 + \frac{a^2}{r^2}\right) \tag{A8}$$

$$\sigma_r^{s2} = \frac{E\delta}{2b} \frac{b^2 - a^2}{c^2 - a^2}\left(1 - \frac{c^2}{r^2}\right) \tag{A9}$$

$$\sigma_\theta^{s2} = \frac{E\delta}{2b} \frac{b^2 - a^2}{c^2 - a^2}\left(1 + \frac{c^2}{r^2}\right) \tag{A10}$$

$$\sigma_r^{A} = \sigma_r^{p} + \sigma_r^{e} + \sigma_r^{s} \tag{A11}$$

$$\sigma_\theta^{A} = \sigma_\theta^{p} + \sigma_\theta^{e} + \sigma_\theta^{s} \tag{A12}$$

Here, the superscript 'A' is the residual stress by autofrettage.

$$\sigma_r^{L1} = \frac{2}{\sqrt{3}} A_1 \ln\left(\frac{r}{a}\right) + \frac{\sigma_y - A_1}{\sqrt{3}B_1} \rho_L^{2B_1}\left(\frac{1}{a^{2B_1}} - \frac{1}{r^{2B_1}}\right) - p_a \tag{A13}$$

$$\sigma_\theta^{L1} = \frac{2}{\sqrt{3}} A_1 \left[ \ln \frac{r}{a} + 1 \right] + \frac{\sigma_y - A_1}{\sqrt{3}B_1} \rho_L^{2B1}\left[\frac{1}{a^{2B_1}} + (2B_1 - 1)\frac{1}{r^{2B_1}}\right] - p_a \tag{A14}$$

$$\sigma_r^{L2} = \frac{\sigma_y}{\sqrt{3}} \rho_L^2 \left(\frac{1}{c^2} - \frac{1}{r^2}\right) \tag{A15}$$

$$\sigma_\theta^{L2} = \frac{\sigma_y}{\sqrt{3}} \rho_L^2 \left(\frac{1}{c^2} + \frac{1}{r^2}\right) \tag{A16}$$

$$\sigma_r^{U1} = \frac{2}{\sqrt{3}} A_3 \ln\left(\frac{r}{a}\right) + \frac{\sigma_D - A_3}{\sqrt{3}B_2} \rho_U^{2B2}\left(\frac{1}{a^{2B_2}} - \frac{1}{r^{2B_2}}\right) - p_a \tag{A17}$$
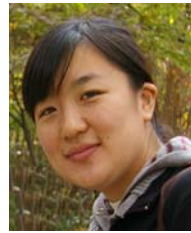
$$\sigma_\theta^{U1} = \frac{2}{\sqrt{3}} A_3 \left[ \ln\left(\frac{r}{a}\right) + 1 \right] + \frac{\sigma_D - A_3}{\sqrt{3}B_2} \rho_U^{2B2}\left[\frac{1}{a^{2B_2}} + (2B_2 - 1)\frac{1}{r^{2B_2}}\right] - p_a \tag{A18}$$

$$\sigma_r^{U2} = \frac{\sigma_D}{\sqrt{3}} \rho_U^2 \left(\frac{1}{c^2} - \frac{1}{r^2}\right) \tag{A19}$$

$$\sigma_\theta^{U2} = \frac{\sigma_D}{\sqrt{3}} \rho_U^2 \left(\frac{1}{c^2} + \frac{1}{r^2}\right) \tag{A20}$$

$$\sigma_r^{A} = \sigma_r^{L} - \sigma_r^{U} + \sigma_r^{S} \tag{A21}$$

$$\sigma_\theta^{A} = \sigma_\theta^{L} - \sigma_\theta^{U} + \sigma_\theta^{S} \tag{A22}$$

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